

Lecture Guide

Math 42 - Elementary Algebra

to accompany
Introductory Algebra, 3rd edition

Miller, O'Neill, Hyde

Prepared by

Stephen Toner

Victor Valley College

Accompanying videos can be found at www.MathVideos.Net.

Last updated: 6/7/14

1.1 - Sets of Numbers and the Real Number Line

A. Types of Numbers

Natural -

Whole -

Integers –

Classify each number below as an integer or not.

	Integer?	
	Yes	No
20		
$\frac{18}{9}$		
$-\frac{7}{9}$		
67		
-55.68		

Rational - any number which can be expressed as the _____ of two numbers, provided the denominator is not zero.

Irrational -

Classify each number below as irrational or rational

	Rational	Irrational
$-\sqrt{64}$		
$-\frac{2}{5}$		
$\sqrt{7}$		
$-99.\overline{46}$		
-9π		

Real –

B. Inequalities

Symbols:

less than

less than or equal to

greater than

greater than or equal to

not equal to

*Insert the proper inequality: 11 _____ 13

*True or False? ... $8 \leq 8$

C. Opposites and Absolute Values

The opposite of 8 is _____.

The opposite of -15 is _____.

The opposite of x is _____.

The **absolute value** of a number is its _____ from zero on the number line.

$|-3| =$ _____

$|14| =$ _____

$|-3 + 7| =$ _____

1.2 – Algebraic Operations

A. Words for Operations

+	−	×	÷

*Write each using algebraic notation:

Six more than a number

Six less than a number

Six less a number

Four more than a number squared

Four more than a number, squared

B. Evaluating Algebraic Expressions

*Evaluate each if $x = 3$ and $y = 2$:

$$4x - 3y$$

$$2x^2 - 5x + 11$$

C. Exponents

x^5 is pronounced "x to the fifth power"

x is called the _____.

5 is called the _____.

$$3^2$$

$$4^3$$

D. Square Roots

$$\sqrt{16} = \text{_____} \text{ since } \text{_____}.$$

$$\sqrt{49} =$$

$$\sqrt{144} =$$

By definition, the square root of a number must be positive. $\sqrt{x^2} = \text{_____}$

***Note that $\sqrt{25} \neq -5$ even though $(-5)^2 = 25$.

E. Order of Operations

Please Excuse My Dear Aunt Sally

P E M A
 D S

A note about notation:

a) Different Uses of Parentheses

$8(5)$ multiplication

$(8 + 5) - (2 + 7)$ grouping

b) Algebra Notation

$5x$ $5 \cdot x$ $5(x)$

$1\frac{1}{2}$ means $1 + \frac{1}{2}$

1.2 #56

$$\left(\frac{9}{8} - \frac{1}{3}\right) \cdot \frac{3}{4}$$

1.2 #64

$$3^2 + \sqrt{16} \cdot 2$$

1.2 #71

$$48 - 13 \cdot 3 + [(50 - 7 \cdot 5) + 2]$$

1.2 #69

$$[4^2 \cdot (6 - 4) \div 8] + [7 \cdot (8 - 3)]$$

1.3 & 1.4 - Adding and Subtracting Signed Numbers

The "Rules" for Addition:

$$9 - (-8)$$

$$-3 - (-5)$$

$$-11 - 4$$

$$14 - (+7)$$

Exercises:

$$-8 + (-7)$$

$$14 + (-9)$$

$$-8 + (3)$$

$$\frac{-1}{4} + \frac{2}{3}$$

$$-7.2 - 3.5$$

$$-14.2 - 5.6$$

$$-13 + (-9)$$

$$4.1 + (-3.2)$$

Technically, all subtraction must be changed into **addition of the opposite**.

Exercises:

$$7 - (-3)$$

1.5 - Multiplying & Dividing Signed Numbers

The "Rules" for Multiplying and Dividing:

Exercises:

$40 \div (-8)$

$7(-8)$

$(-3)(-9)$

$\frac{-20}{4}$

$(-2)(3)(-4)(2)$

$(-1)(-1)(-1)(-1) \cdots (-1) \quad (57 \text{ of them})$



Property: $(-1)^n = \begin{cases} +1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd} \end{cases}$

Be careful! -2^4 versus $(-2)^4$

*Evaluate each if $x = -3$ and $y = -2$:

a. $x^2 - y^2$

b. $2x^2 - 5y$

c. $\frac{2x-y}{x+y}$

1.5 #112 Order of Operations

$$-6 - 5[-4 - (6 - 12)] + (-5)^2$$

1.5 #116 Order of Operations

$$\frac{|-2 + 6| - |3 - 5|}{13 - 11}$$

Mixed Review

1. $(-3) - 9$

2. -2^2

3. $4 - (-7)$

4. $-3 + 8$

5. $5 - 2^2$

6. $(-4)(-2)$

7. $10 \div (-2)$

8. $(-5)^2$

9. $14 + (-3)$

10. $4 - 3$

11. $4 - (-3)$

12. $100 - 47$

13. $-18 - (+22)$

14. $4^2 - 3^2$

15. $8 + (-8)$

16. $3 - (-3)$

17. $0 \div (-7)$

18. $(-18) \div 0$

19. $|-4 - 6| - |2 + (-3)|$

20. $\frac{10}{-5} + (-2)(-4) - 5$

1.6 - Properties of Real Numbers and Simplifying Expressions

A. Properties

Commutative Property...

... of addition

... of multiplication

Associative Property...

... of addition

... of multiplication

Additive Identity

Multiplicative Identity

Additive Inverse

Multiplicative Inverse

Distributive Properties

B. Expressions

Terms are separated by + or - signs.

When combining like terms, add their coefficients, but keep their variable parts the same.

* Combine like terms / simplify:

a. $4x + 3x + x$

b. $5x + 3y - 2x + 4$

c. $3(2x - 4) + 11$

d. $-4(-2x + 3) - 5(4x + 7)$

2.1 - Solving Linear Equations, Part 1 - One-Step Equations

We use the directions "solve" when working with equations. Casually speaking, it means "find the value of x which 'works' in the equation."

*Identify the following as either an expression or an equation.

1. $x + 6 = 15$

2. $4x - 9$

3. $4x^2 - 5x + 3 = 0$

*Determine whether the given number is a solution to the equation.

4. $3x = 24$; 8

5. $2x - 3 = 17$; 7

6. $2x - 5 = -11$; -3

*Solve (and check) each of the following equations.

7. $x + 8 = 14$

8. $m - 14 = 22$

9. $-6 = c + 13$

10. $W - \frac{2}{3} = \frac{11}{3}$

11. $3x = 63$

12. $24 = -8q$

13. $\frac{3}{5}x = 15$

14. $-2.4r = -7.2$

15. $\frac{x}{5} = 10$

16. $\frac{-1}{3}x = 4$

*Write an algebraic equation to represent each English sentence. Then solve the equation.

17. The sum of twenty-five and a number is twelve.

18. The difference of a number and negative seven is sixty-three.

19. The product of five and a number is thirty.

20. The quotient of a number and nine is one-fourth.

2.2 - Solving Linear Equations, Part 2 - Multi-step equations

* Solve each of the following equations.

1. $3y - 8 = 22$

2. $-4p + 7 = -5$

3. $0.2x + 5.3 = -2.6$

4. $\frac{3}{8} = \frac{1}{2} - \frac{1}{4}W$

5. $6d + d - 3 = 15$

6. $8x - 5x + 3 = 21 - 6$

7. $-4h + 9 = 7h + 6$

8. $8 - 3v = 7v + 14$

9. $\frac{2}{3}p + \frac{1}{4} = \frac{1}{6}p + \frac{1}{2}$

10. $5(2x + 7) = 15$

11. $14(3c + 6) + 4(2c + 7) = 14$

2.2 #50

$$-5y + 2(2y + 1) = 2(5y - 1) - 7$$

2.2 #56

$$5 - (6k + 1) = 2[(5k - 3) - (k - 2)]$$

*Identify the equation as a conditional equation, a contradiction, or an identity. Then describe the solution.

1. $5x + 3 = 5(x + 2)$

2. $7x - 2 = x + 6(x + 1) - 8$

3. $4x - 2(3 - x) = 10 + 2x$

Mixed Review. Solve each equation.

a. $q - \frac{1}{5} = \frac{2}{3}$

b. $10 - 3q = 25$

2.2 #92 $6w - (8 + 2w) = 2(w - 4)$

2.2 #85 $10(2n + 1) - 6 = 20(n - 1) + 12$

2.3 - Solving Linear Equations, Part 2 - Clearing Fractions and Decimals

*Solve the following equations.

$$2.3 \text{ \#18 } \frac{2}{15}z + 3 = \frac{7}{5}$$

$$2.3 \text{ \#30 } \frac{3}{2}b - 1 = \frac{1}{8}(12b - 8)$$

$$2.3 \text{ \#36 } \frac{5y-1}{2} - \frac{y+4}{5} = 1$$

$$2.3 \text{ \#38 } -6.3x + 1.5 = -4.8$$

$$2.3 \text{ \#58 } \frac{2}{3}(y - 1) = \frac{3}{4}(3y - 2)$$

$$\text{Solve: } \frac{3x}{5} + 2 = \frac{3x}{4} - 7$$

2.4 - Applications of Linear Equations

A. "Read and Write" Word Problems

2.4 #10 Fifteen less than a number is 41. Find the number.

2.4 #14 Three times a number is the same as the difference of twice the number and seven. Find the number.

2.4 #16 The difference of 14 and three times a number is the same as the sum of the number and -10. Find the number.

2.4 #18 Twice the sum of a number and eleven is twenty-two less than three times the number. Find the number.

B. Consecutive Integer Word Problems

Consecutive integers

Consecutive Even Integers

Consecutive Odd Integers

2.4 #22 The sum of two consecutive odd integers is 52. Find the integers.

2.4 #26 The perimeter of a triangle is 96 in. The lengths of the sides are represented by consecutive integers. Find the measures of the sides.

2.4 #28 The sum of three consecutive odd integers is 89 less than twice the largest integer. Find the integers.

2.4 #30 Five times the sum of three consecutive even integers is 140 more than ten times the smallest. Find the integers.

C. Other Word Problems

2.4 #32 A rope is 54 in. in length and must be cut into two pieces. If one piece must be twice as long as the other, find the length of each piece.

2.4 #34 Maria's age is 15 years less than Orlando's age. The sum of their ages is 29. Find their ages.

2.4 #44 \$120 is to be split among three restaurant servers. Angie made \$10 more than Marie. Gwen, who went home sick, made \$25 less than Marie. How much money should each server get?

2.5 - Applications Involving Percents

A. Percent Sentence Problems

1. What is 36% of 400?
2. 40 is 20% of what number?
3. What percent of 80 is 16?

B. Simple Interest Problems

$$I = Prt$$

$$A = P + Prt$$

2.5 #26 How much interest will Roxanne have to pay if she borrows \$2000 for 2 yr at a simple interest rate of 4%?

2.5 #28 Andrea borrowed some money for 2 yr at 6% simple interest. If she had to pay back a total of \$3640, how much did she originally borrow?

C. Pricing Word Problems

$$\text{Cost} - \text{Discount} = \text{Sale Price}$$

$$\text{Wholesale} + \text{Markup} = \text{Selling Price}$$

2.5 #36 The *Avatar* DVD is on sale for \$18. If this represents an 18% discount rate, what was the original price of the DVD?

2.6 - Formulas and Applications of Geometry

A. Literal Equations

*Solve for the indicated variable.

1. $x + y = -2$ for y .

2. $I = Prt$ for t .

3. $2l + 2w = P$ for w .

4. $Q = \frac{x+y}{2}$ for y .

5. $3x + 5x = q + r$ for x .

B. Using Formulas

$$A = l \cdot w$$

$$A = s^2$$

$$P = 2l + 2w$$

$$P = 4 \cdot s$$

$$P = a + b + c$$

*Theorem: In triangles, all three angles have measures that add up to _____ degrees.

*Vertical angles:

***Complementary** angles add up to _____ degrees.

***Supplementary** angles add up to _____ degrees.

2.6 #41 The perimeter of a rectangular garden is 24 ft. The length is 2 ft more than the width. Find the length and width of the garden.

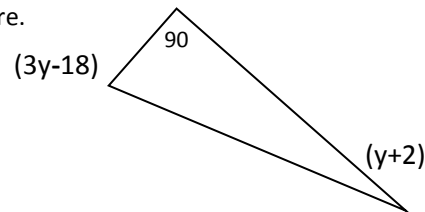
2.6 #44 The width of Jason's workbench is $\frac{1}{2}$ the length. The perimeter is 240 in. Find the length and the width of the workbench.

2.6 #52 Two angles are complementary. One angle is 4° less than three times the other angle. Find the measure of the angles.

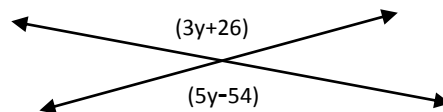
2.6 #54 Two angles are supplementary. One angle is 6° more than four times the other. Find the measures of the two angles.

2.6 #58 The smallest angle in a triangle measures 90° less than the largest angle. The middle angle measures 60° less than the largest angle. Find the measure of each angle.

2.6 #62 Find the value of y and the measure of each angle labeled in the figure.



2.6 #56 Find the measures of the vertical angles labeled in the figure by first solving for y .



2.7 – Mixture and Motion Word Problems

A. Mixture Problems

2.7 #24 How many ounces of a 50% antifreeze solution must be mixed with 10 oz of an 80% antifreeze solution to produce a 60% antifreeze solution?

2.7 #26 A landscaper needs to mix a 75% pesticide solution with 30 gal of a 25% pesticide solution to get a 60% pesticide solution. How many gallons of the 75% solution must he use?

2.7 #14 Anna is a teacher at an elementary school. She purchased 72 tickets to take the first-grade children and some parents on a field trip to the zoo. She purchased children's tickets for \$10 each and adults' tickets for \$18 each. She spent a total of \$856. How many of each ticket did she buy?

2.7 #18 Mr. Garvey purchased 58 food items at a local fast food restaurant for his little league team. He purchased hamburgers for \$2.50 each and french fries for \$1.50 each. He spent a total of \$127. How many hamburgers and how many french fries did he purchase?

B. Motion Problems

2.7 #32 A car travels 20 mph slower in a bad rain storm than in sunny weather. The car travels the same distance in 2 hr in sunny weather as it does in 3 hr in rainy weather. Find the speed of the car in sunny weather.

2.7 #38 Two cars are 238 miles apart and traveling toward each other along the same road. They meet in 2 hr. One car is traveling 5 mph slower than the other. What is the speed of each car?

A car passes a landmark on a highway traveling at a constant rate of 40 kilometers per hour. An hour and a half later, a second car passes the same landmark traveling in the same direction at 70 kilometers per hour. How much time after the second car passes the landmark will it overtake the first car?

2.7 #48 An air traffic controller observes a plane heading from New York to San Francisco traveling at 450 mph. At the same time, another plane leaves San Francisco and travels 500 mph to New York. If the distance between the airports is 2850 mi, how long will it take for the planes to pass each other?

2.8 – Solving Inequalities

Inequality Graph Interval Notation

*Solve each. Graph your solution. Also write your solution using interval notation.

1. $x + 5 \geq 11$ 

2. $3x - 7 < 19$ 

Exception to the rule: Whenever you multiply or divide each side of the inequality by a negative, then you must change the direction of the inequality.

c. $5 - 2x \leq 17$ 

d. $-17 < 2x - 7 \leq 15$ 

e. $-5x < 30$ 

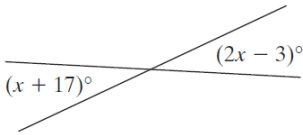
Additional Word Problems

2.7 #36 A boat on the Potomac River travels the same distance downstream in $\frac{2}{3}$ hr as it does going upstream in 1 hr. if the speed of the current is 3 mph, find the speed of the boat in still water.

2.4 #48 Four times the smallest of 3 consecutive odd integers is 236 more than the sum of the other two. Find the three integers.

2.7 #25 A pharmacist needs to mix a 1% saline (salt) solution with 24 milliliters (mL) of a 16% saline solution to obtain a 9% saline solution. How many milliliters of the 1% solution must she use?

2.6 #55 Find the measures of the vertical angles labeled in the figure by first solving for x.



2.4 #62 Find the value of y and the measure of each angle labeled in the figure.

