

Lecture Guide

Math 90 - Intermediate Algebra

to accompany

Intermediate Algebra, 3rd edition

Miller, O'Neill, & Hyde

Prepared by

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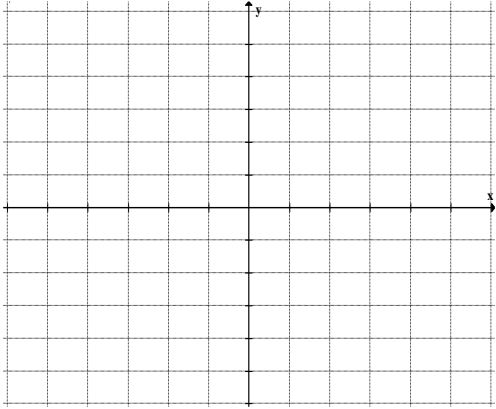
Victor Valley College

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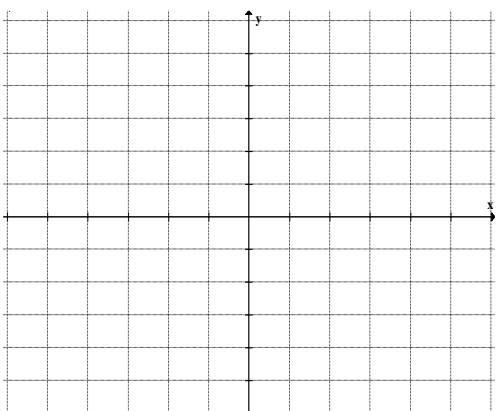
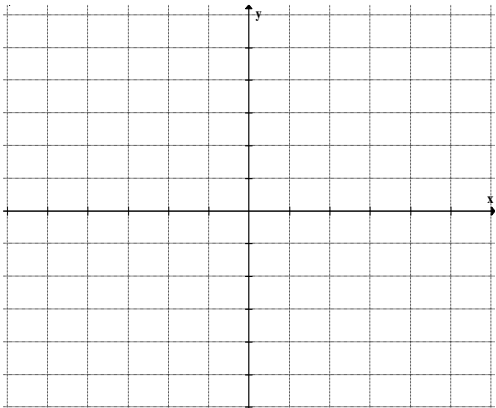
2.1 – The Rectangular Coordinate System

Introduction to Graphing

The four quadrants:



(x,y) coordinates:

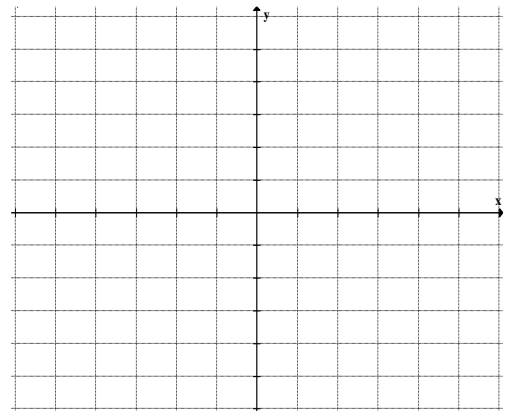


A line is an infinite collection of (x,y) points, each of which satisfy the equation of that line.

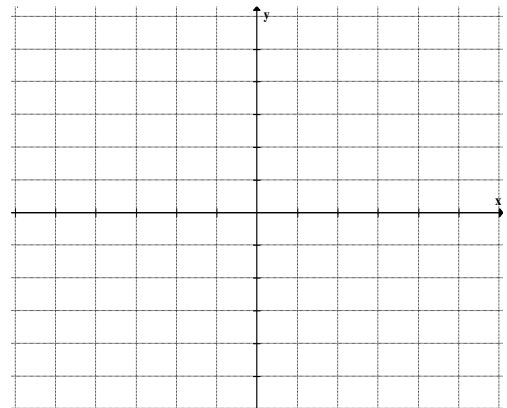
*Is $(3, -4)$ on the line $6x + 5y = -2$?

*Graph each line:

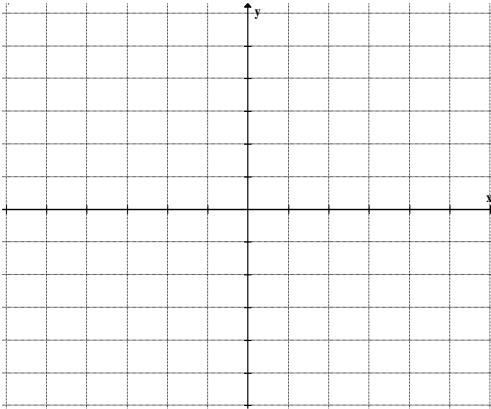
1. $y = x + 5$



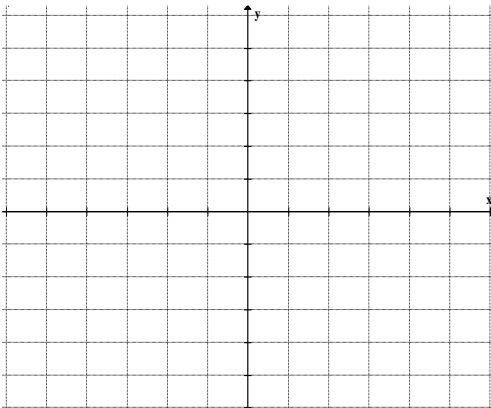
2. $y = -x + 2$



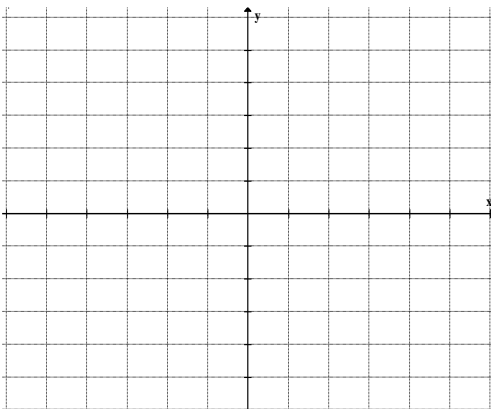
3. $y = 2x + 1$



4. $4x + 3y = 12$

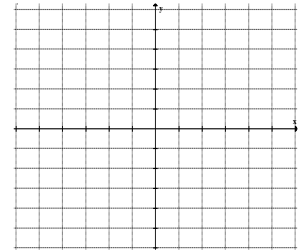
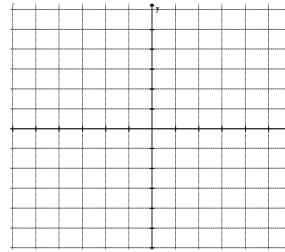


5. $2x - 3y = 6$

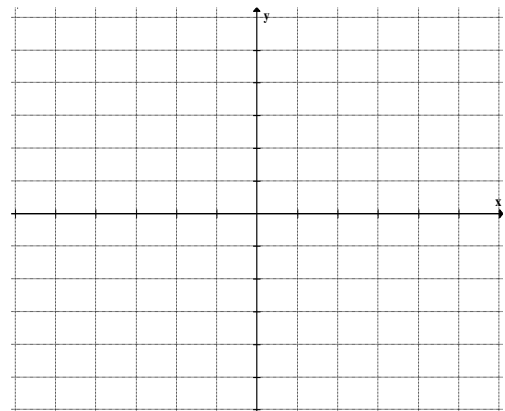


Summary:

- When the given line is of the form $y = \underline{\hspace{1cm}}$, use an x-y chart with any choices of x (usually $x=0,1,2$ is best).
- When the given line is of the form $ax + by = c$, also known as standard form, graph using intercepts.
- When the given line is missing a variable, it is either horizontal or vertical.



6. Graph $y = \frac{1}{2}x + 3$

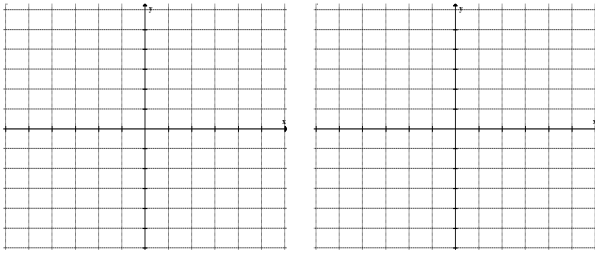


7. Find the x- and y-intercepts of $4x - 7y = 16$.

2.2 – Slope

Formula: $m = \frac{\text{rise}}{\text{run}} =$

*Find the slope of each line:

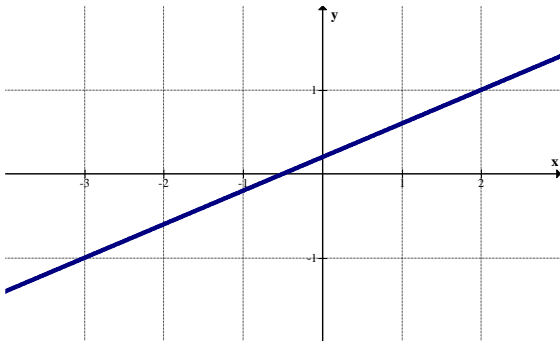


*Find the slope of the line which runs through the given pair of points:

1. $(-2,3)$ and $(4,1)$

2. $(4,5)$ and $(11,-3)$

3. Find the slope of the graphed line (passes through $(-3,-1)$ and $(2,1)$):



**Parallel lines have the _____ slope.

**The slopes of perpendicular lines are

_____ of each other.

Without graphing, determine whether the lines through the given pairs of points are parallel, perpendicular, or neither.

4. $L_1 : (-3, -5)$ and $(-1, 2)$
 $L_2 : (0, 4)$ and $(7, 2)$

2.3 – Slope Intercept and Point-Slope Forms

*In each equation, identify the slope and the y-intercept.

1. $y = 3x - 5$

2. $y = \frac{1}{3}x - \frac{4}{7}$

3. $y = 5x$

4. $y = x - 9$

5. $3x + 7y = 11$

6. $2x + 5y = 3$

7. $x = 7$

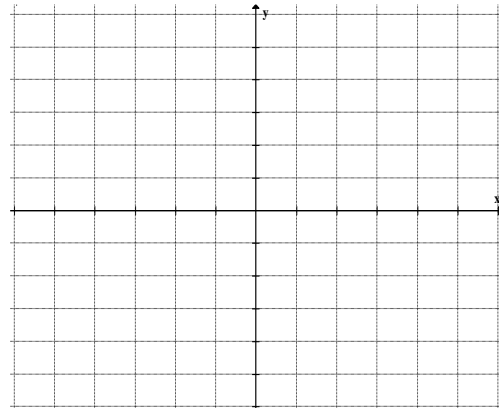
*Are the following pairs of lines parallel, perpendicular, or neither?

8.
$$\begin{cases} y = \frac{3}{5}x + 2 \\ y = \frac{-3}{5}x + 7 \end{cases}$$

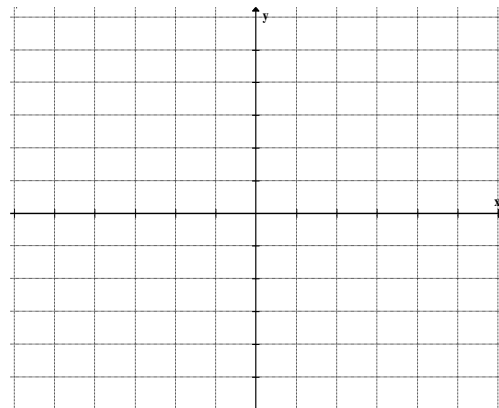
9.
$$\begin{cases} y = x + 4 \\ y = -x + 7 \end{cases}$$

10.
$$\begin{cases} y = \frac{2}{3}x + 4 \\ y = \frac{2}{3}x + 7 \end{cases}$$

11. Graph: $y = \frac{-3}{5}x + 2$



12. Graph: $y = \frac{1}{4}x - 3$



Three Forms of a Line

1. $ax + by = c$... standard form
2. $y = mx + b$... slope-intercept form
3. $y - y_1 = m(x - x_1)$... point-slope form

*Find the equation of the line (in slope-intercept form) which has the following characteristics.

a. $m = \frac{2}{3}$ and has y-intercept (0,5).

b. $m = \frac{-3}{5}$ and the line passes through (-1,4).

c. passes through (-2,5) and (3,7).

d. passes through (-5,1) and is parallel to the line $3x - 4y = 11$.

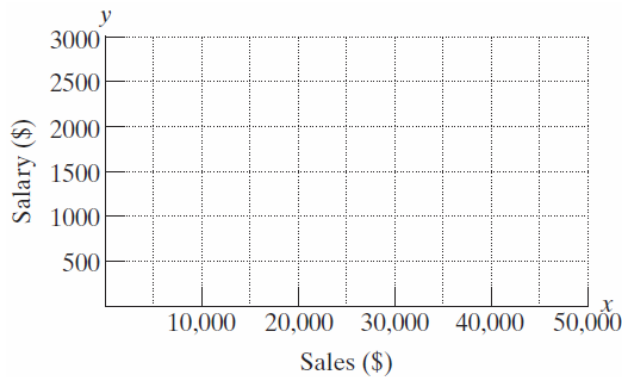
e. passes through (-6,2) and is perpendicular to the line $y = \frac{-4}{3}x + 7$.

2.4 – Linear Models

Alex is a sales representative and earns a base salary of \$1000 per month plus a 4% commission on his sales for the month.

a. Write a linear equation that expresses Alex's monthly salary y in terms of his sales x .

b. Graph the equation.

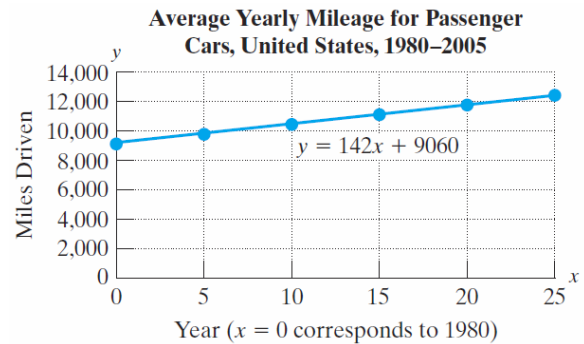


c. What is the y -intercept and what does it represent in the context of this problem?

d. What is the slope of the line and what does it represent in the context of this problem?

e. How much will Alex make if his sales for a given month are \$30,000?

Let y represent the average number of miles driven per year for passenger cars in the United States since 1980. Let $x=0$ represent the year where corresponds to 1980, $x=1$ corresponds to 1981, and so on. The average yearly mileage for passenger cars can be approximated by the equation $y=142x+9060$ where $x \geq 0$.



a. Use the linear equation to approximate the average yearly mileage for passenger cars in the United States in the year 2005.

b. Use the linear equation to approximate the average mileage for the year 1985, and compare it with the actual value of 9700 mi.

c. What is the slope of the line and what does it mean in the context of this problem?

d. What is the y -intercept and what does it mean in the context of this problem?

3.1 – Relations

The **domain** of an expression is the set of values which _____ substituted into the expression.

The **range** of an expression is the set of values which can _____ the expression.

Example: $y = x^2$

domain:

range:

To find the domain, start with a "default"

domain of _____

and then take away x-values which...

**

**

**

*Find the domain of each:

$$f(x) = \frac{x + 2}{x - 3}$$

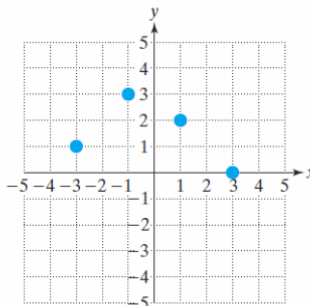
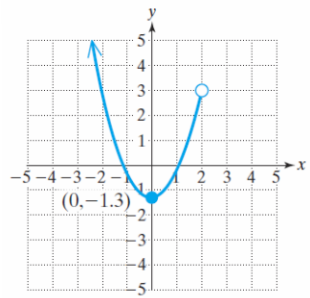
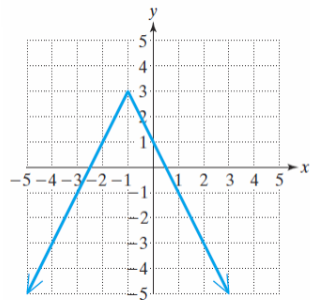
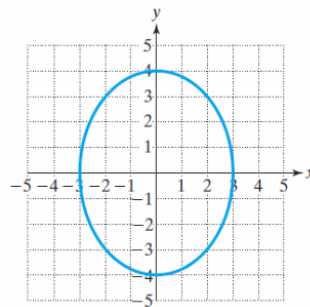
$$g(x) = \sqrt{x - 3}$$

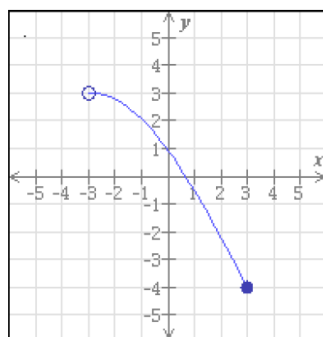
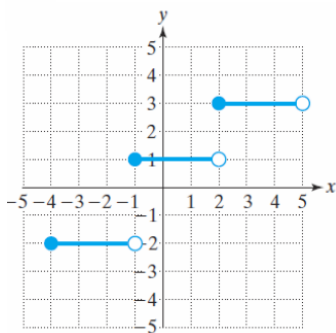
$$f(x) = \frac{x + 7}{x^2 - x - 6}$$

To determine the domain from a graph, look at where the graph extends, left to right.

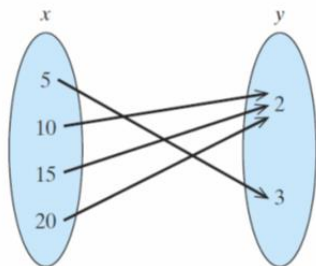
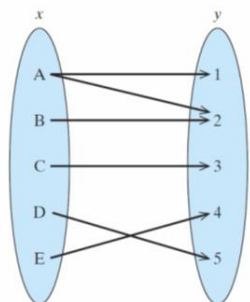
To determine the range from a graph, look at where the graph extends vertically.

*Find the domain and range of each graph:





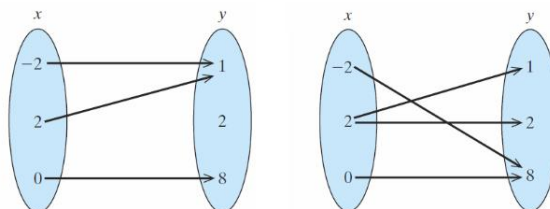
*Find the domain and range of each:



3.2 – Functions

A function is a rule which assigns a _____ y-value in the range to each x value in its domain.

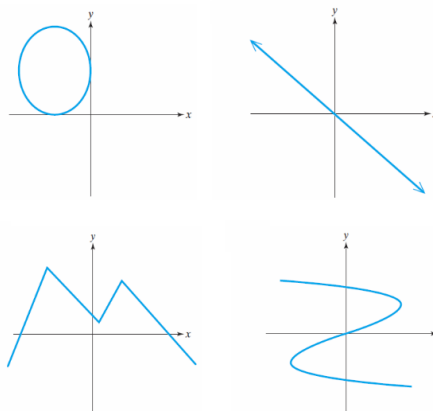
*Do the following relations represent functions?



$\{(1, 2), (3, 4), (5, 4), (-9, 3)\}$

A graph is that of a function if it passes the _____

*Are the following graphs those of functions?



Notation: $f(x)$ is pronounced _____

Anytime you see $f(x)$, you can replace it with y .

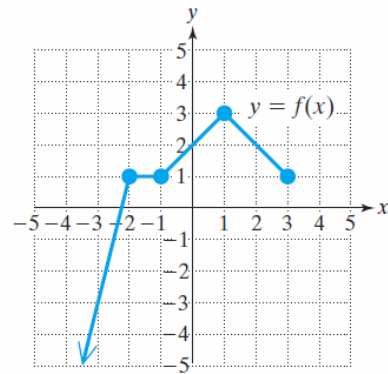
When we write $f(2) = 3$, we mean _____

Given, $\begin{cases} f(x) = 2x^2 + 3 \\ g(x) = 5x + 7 \\ h(x) = \frac{1}{2}x - 4 \end{cases}$, find the following:

- $g(-3) =$
- $h(10) =$
- $f(-3) =$
- $g(x + 1) =$
- $f(w) =$
- $g(\odot) =$
- $h(4) + f(1) =$
- $5 \cdot g(2)$

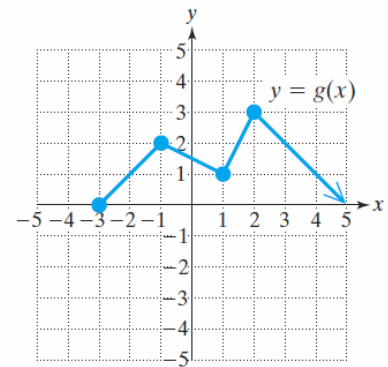
*Find the requested values from the graph:

The graph of $f(x)$ is given.



- Find $f(0)$.
- Find $f(3)$.
- Find $f(-2)$.
- For what value(s) of x is $f(x) = -3$?
- For what value(s) of x is $f(x) = 3$?
- Write the domain of $f(x)$.
- Write the range of $f(x)$.

The graph of $g(x)$ is given.



- Find $g(-1)$.
- Find $g(1)$.
- Find $g(4)$.
- For what value(s) of x is $g(x) = 3$?
- For what value(s) of x is $g(x) = 0$?
- Write the domain of $g(x)$.
- Write the range of $g(x)$.

*Find the domain of each function. Write the answers in interval notation.

$$m(x) = \frac{x - 1}{x - 4}$$

$$f(t) = \frac{5}{t}$$

$$n(p) = \frac{p + 8}{p^2 + 2}$$

$$k(t) = \sqrt{t - 5}$$

$$m(x) = \sqrt{1 - 2x}$$

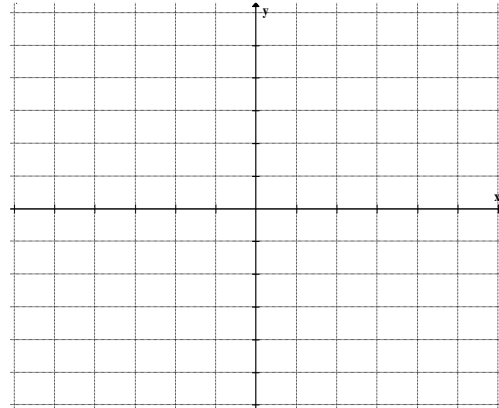
$$q(t) = t^3 + t - 1$$

$$h(x) = \frac{1}{\sqrt{x - 5}}$$

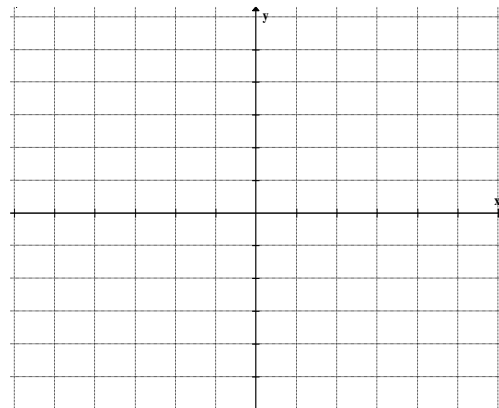
3.3 – Graphs of Basic Functions

Graph each of the following. Then state the domain and range of each.

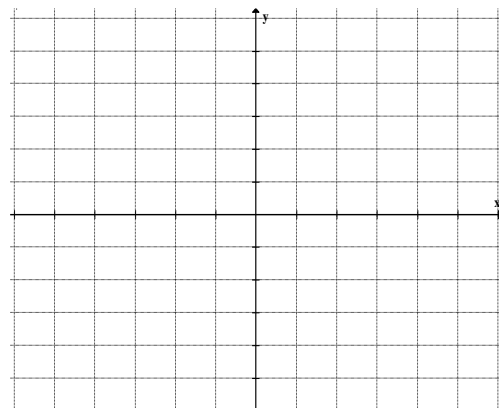
1. $f(x) = x$ Linear Function



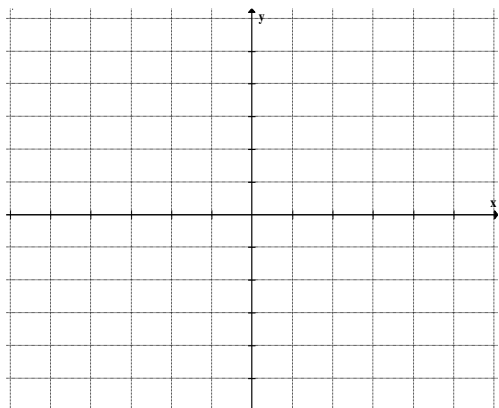
2. $f(x) = x^2$ Quadratic Function



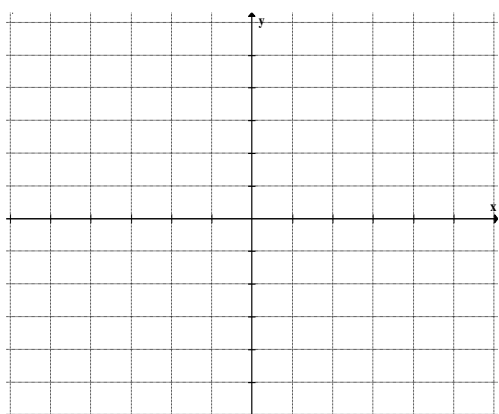
3. $f(x) = x^3$ Cubic Function



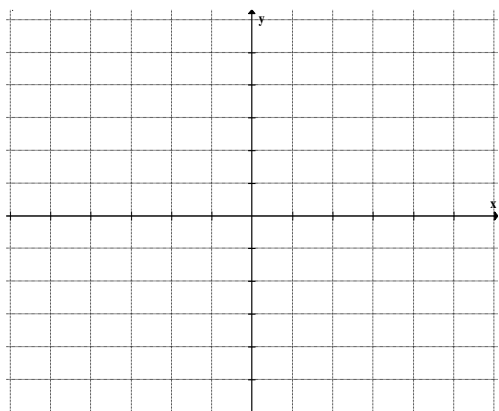
4. $f(x) = |x|$ Absolute Value Function



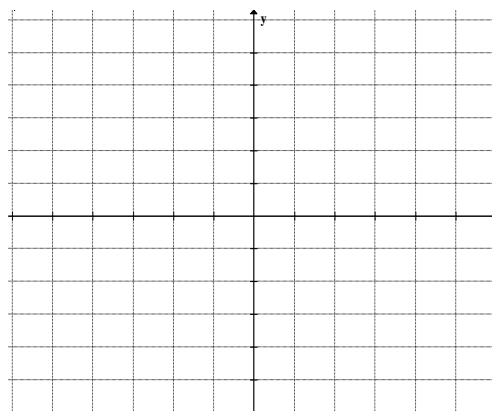
5. $f(x) = \sqrt{x}$ Square Root Function



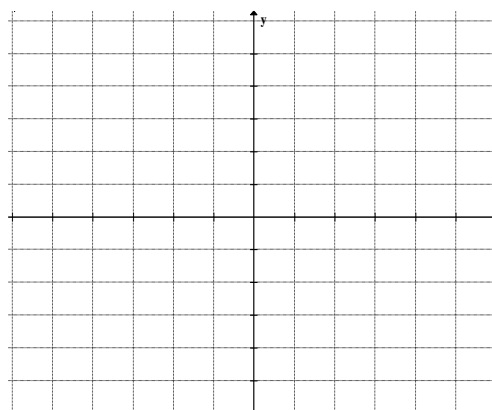
6. $f(x) = \frac{1}{x}$ Reciprocal Function



Graph the parabola $y = \frac{5}{3}x^2$. Indicate five points on its graph.



Graph the cubic function $y = \frac{-3}{4}x^3$. Indicate three points on its graph.



3.4 – The Algebra of Functions

1. Given $\begin{cases} f(x) = x^2 + 3 \\ g(x) = x - 2 \end{cases}$, find the following:

a. $(f + g)(x)$

b. $(f - g)(x)$

c. $(f \cdot g)(-2)$

d. $f \circ g(x)$

e. $g \circ f(x)$

f. $f \circ g(3)$

2. Given $\begin{cases} f(x) = \sqrt{x + 1} \\ g(x) = 2x - 5 \\ h(x) = x^2 - 3 \end{cases}$, find the following:

a. $f \circ g(x)$

b. $h \circ f(x)$

c. $g \circ h(5)$

d. $g \circ g(3)$

The domains of $f + g$, $f - g$ and fg will **all be the same** (the intersection of their separate domains). The domain of $\frac{f}{g}$ will be further restricted so that $g(x) \neq 0$.

To find the domains of **composite** functions, compose them and then analyze the function that results.

3. Given $f(x) = \sqrt{x-2}$ and $g(x) = x-4$, find...

a. the domain of $f(x)$.

b. the domain of $g(x)$.

c. the domain of $f + g$, $f - g$ and fg .

d. the domain of $\frac{f}{g}(x)$.

4. Given $f(x) = \frac{x+3}{x-4}$ and $g(x) = \frac{x-1}{x+2}$, find...

a. the domain of $f(x)$.

b. the domain of $g(x)$.

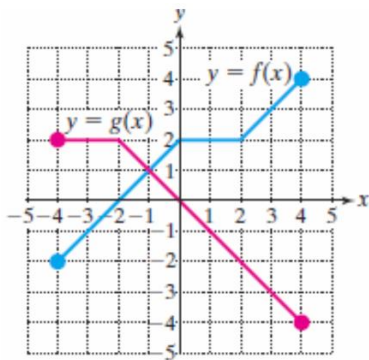
c. the domain of $f + g$, $f - g$ and fg .

d. the domain of $\frac{f}{g}(x)$.

To find the domain of a composite function, find the composition and then analyze it.

5. Given $f(x) = \sqrt{x-3}$ and $g(x) = x+7$, find the domain of $f \circ g(x)$.

Use the graphs to find the function values:

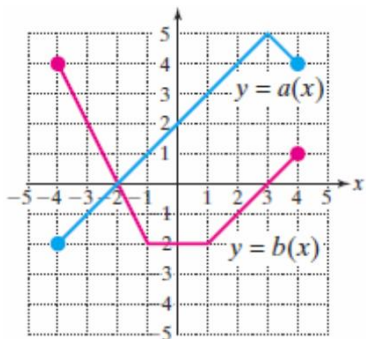


1. $f(-4)$ 2. $f(1)$ 3. $g(-2)$ 4. $g(3)$ 5. $(f + g)(2)$

6. $(g - f)(3)$ 7. $(f \cdot g)(-1)$ 8. $(g \cdot f)(-4)$

9. $\left(\frac{g}{f}\right)(0)$ 10. $\left(\frac{f}{g}\right)(-2)$ 11. $\left(\frac{f}{g}\right)(0)$ 12. $\left(\frac{g}{f}\right)(-2)$

13. $g \circ f(-1)$ 14. $f \circ g(0)$ 15. $f \circ g(-4)$ 16. $g \circ f(-4)$ 17. $g \circ g(2)$ 18. $f \circ f(-2)$



19. $a(-3)$ 20. $a(1)$ 21. $b(-1)$ 22. $b(3)$

23. $(a - b)(-1)$ 24. $(a + b)(0)$ 25. $(b \cdot a)(1)$ 26. $(a \cdot b)(2)$

27. $b \circ a(0)$ 28. $a \circ b(-2)$ 29. $a \circ b(-4)$ 30. $b \circ a(-3)$

31. $\left(\frac{b}{a}\right)(3)$ 32. $\left(\frac{a}{b}\right)(4)$ 33. $a \circ a(-2)$

79. The cost in dollars of producing x toy cars is $C(x) = 2.2x + 1$. The revenue received is $R(x) = 5.98x$. To calculate profit, subtract the cost from the revenue.

a. Write and simplify a function P that represents profit in terms of x .

b. Find the profit of producing 50 toy cars.